

THE METHOD OF THE FIELD OF THERMAL
CHARACTERISTICS FOR DETERMINING THE
TEMPERATURE FIELD IN DISPERSE SYSTEMS

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A concept is described from which it is possible to formulate an effective heat conduction equation in which the parameters are functions of the properties and the state of the material. A method is proposed for the solution of similar problems, noting the difficulties which occur at each stage of the similar approaches.

In the course of many years the author has solved a variety of problems in the determination of the temporal and spatial distribution of temperatures in the most varied disperse objects (soil, physiological solutions and compounds, semiconductors and dielectric compositions having in addition adsorption qualities), and also in other varieties of structures of a disperse nature.

The most common disperse material is a two-phase (dry) or a three-phase (moist) system: a framework or skeleton comprising a collection of large numbers of solid particles of various shapes and sizes separated from each other by gaps which are filled with gas, moisture, or both simultaneously. In other cases this framework can be a gas or a liquid containing solid particles of extremely small dimension, down to 10^{-7} cm.

If we study only commercially important disperse materials we shall be dealing in the first class with powdered, granular, scaly, and fibrous materials which are widely used in engineering practice. A characteristic of the objects just enumerated, which should be called capillary-porous substances, is the comparatively large dimension of the particles from which the solid framework of the system is constructed (from 0.1 mm upwards).

Building materials, heat insulators, soils and earths, coal, nutritives, etc., for a large part consist of a solid base, the particles of which are measured in tenths of a millimeter or in whole millimeters.

Noting the multiphase nature of capillary-porous substances, we have to imagine that the following mechanisms for heat transfer operate in them:

- 1) thermal conductivity along the separate particles or elements of the solid skeleton of the material;
- 2) heat transfer due to thermal conductivity from one particle to its neighbor where they are in direct contact;
- 3) molecular heat conduction in the medium filling the gaps between the particles;
- 4) heat transfer at the boundary between the solid particles and the medium;
- 5) radiation from one particle to another;
- 6) convection of the gas and moisture between the particles.

All these processes can be reduced to four types of process: 1) heat conduction; 2) convection of the gas or liquid; 3) radiation in the pores; 4) transport of moisture by the liquid.

The determination of the temperature field in a material in which all these factors operate is a problem of exceptional complexity and a single rule for its solution would be the construction and solution, for

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each actual case, of a system of four initial equations: heat transport by conduction, convection, radiation and moisture.

The theory of heat and moisture exchange is at the present time a very strict and fruitful concept which can conveniently be used to solve the above problems on the basis of the theoretical and experimental results obtained by Lykov and his colleagues in the most diverse branches of science and technology [1-3].

Much more restricted, but frequently leading to a positive result, is the author's approach, which he calls the method of the field of thermal characteristics. It should be emphasized that neither in scale, nor in breadth of application, nor in their actual potentialities, can the methods developed by the author, which are essentially rather special, be compared with the universal and widely developed ideas of heat and mass transport. In addition, in no case does the author propose that they be considered in opposition to the theory of heat and mass transport; rather, they are a useful, albeit restricted, complement.

We are discussing here the proposition which must be considered in constructing and solving the equations describing effective heat conduction in a disperse material [4-5]. We have to regard the material as a quasihomogeneous substance to which we can apply the equation of heat conduction, which takes into account the real and specific features, structure, properties, and state of the material. To accommodate the specific characteristics of the material, the thermophysical characteristics in the equations of heat conduction for such multiphase and complex systems are assumed not to be constant, as is usual for solid bodies, but to be functions of the temperature, the time, and the coordinates. These functions must reflect the true relations between the thermophysical characteristics and the properties and state of the substance which occur in the actual conditions. Because radiation, convection, and mass transport occur in disperse systems, these characteristics are effective or equivalent quantities. The essence of our assumption is that in analyzing and determining the temperature field in disperse materials it is possible to use not only the system of equations for conductive, radiative, and mass-transport conduction, but also a single heat conduction equation in which because of the processes mentioned, the coefficients are complex functions of the coordinates, time and temperature

$$c \frac{\partial t}{\partial \tau} = \frac{\partial}{\partial x} \left[\lambda_T(x, \tau, t) \frac{\partial t}{\partial x} \right] + \frac{\partial}{\partial x} \left[\lambda_C(x, \tau, t) \frac{\partial t}{\partial x} \right] + \frac{\partial}{\partial x} \left[\lambda_R(x, \tau, t) \frac{\partial t}{\partial x} \right] + \frac{\partial}{\partial x} \left[\lambda_M(x, \tau, t) \frac{\partial t}{\partial x} \right]. \quad (1)$$

Here c is the volumetric heat and λ is the coefficient of thermal conductivity. One-dimensional (this restriction is assumed only for simplicity of exposition) and unsteady heat propagation is based on the equation

$$c \frac{\partial t}{\partial \tau} = \frac{\partial q}{\partial x}, \quad (2)$$

which states that the heat content of the material is equal to the derivative of the total heat flux q which is the sum of four fluxes q_T , q_C , q_R , and q_M , determined respectively by the conductive heat liberation, convection, radiation, and moisture transport:

$$q = q_T + q_C + q_R + q_M \quad (3)$$

With the familiar expression

$$q = \lambda_T \frac{\partial t}{\partial x} \quad (4)$$

we introduce formally the coefficients of heat conduction by convection (λ_C), radiation (λ_R), and moisture conduction (λ_M) and express the remaining equations in the form (4):

$$q_i = \lambda \frac{\partial t}{\partial x}, \quad i = c, r, m.$$

The solution of each actual problem reduces to an ability on the basis of experiment and physical constructions, to express explicitly the relation between the thermal characteristics λ_T , λ_C , λ_R , and c on the one hand and the temperature, the time, and the coordinates on the other. We substitute the relations obtained in the equation, which we integrate to obtain the solution in the form $t = f(x, \tau, t)$. The greatest difficulties arise at the stage when we have to express the properties and state of the substance in the forms $\lambda(t, x, \tau)$ and $c(t, x, \tau)$. We call this stage the establishment of the field of thermal characteristics.

If the field of thermophysical characteristics is explicit and established, we can, with its help, approach the mastery of the second stage of the problem — the determination of the solution of the equation of heat conduction

with variable characteristics. Although this approach cannot be considered easy, the results achieved by its use in the solution of problems in the thermal conditions of a soil testify in its favour [6, 7]. Of course, these results are achieved after a whole series of simplifications. The fundamental simplifications are: 1) that in soils the variable nature of λ and c , particularly in diurnal processes is determined essentially by the change in the properties and state of the object along the coordinate, the depth, and with time, and does not change as a function of the temperature; 2) that there is a large volume of experimental data for actual soils which makes it possible to express λ_1 and c_1 analytically as explicit functions of x and τ ; 3) that in diurnal processes it is justified to assume that the temperature varies periodically at the surface and at any depth where the diurnal temperature wave is shifted in phase and reduced in amplitude. Thus we can seek the solution in the form of a periodic series and avoid introducing the complicating problems of the initial conditions.

The practical implementation of the method of the field of thermal characteristics in soils was facilitated by the fact that the author and his colleagues have surveyed the fundamental types of soils of the Soviet Union in all regions and Republics which has made it possible to handle the most difficult stage of the problem – the establishment of the field of thermal characteristics – in laboratory and field conditions. Each of the coefficients λ_T , λ_C , λ_R , λ_M is a very complex and varied function of the properties and states of the material. Thus λ_T depends on the structure of the particles, their density and packing, the nature of the contact between them, the temperature, the moisture content and density of the material. There are dozens of equations expressing λ_T for the assumed model of the structure of a disperse system in terms of the moisture and air contents and the ratio of their sum to the total volume of the material.

There are many calculations in which it is proposed to consider a dispersive object as a sequential series of thermal resistances of the solid and intermediate media along the path of the thermal flux by analogy with electrical resistance. Even more frequently we encounter empirical equations obtained from the analysis of statistical data on the correlation between the coefficient of heat conductivity and the various factors affecting it.

To illustrate the complexity of the problem we note that even in the simplest case of a two-phase system, when it is assumed that all the particles have approximately the same dimension, which is far from realistic, the results of calculations on the coefficient of conductive heat conduction for similar systems are greatly lacking in uniformity. Depending on the assumed form of the particles, their packing, the nature of their relative disposition, and the direction of the thermal flux in relation to the planes of the particles, many equations have been proposed (the following notation has been introduced in them: P is the porosity, i. e., the ratio of the air or moisture content to the total volume of the system; λ_0 and λ_i are the coefficients of thermal conductivity of the solid phase and the intermediate medium) of the form

$$\lambda = f(P, \lambda_0, \lambda_i).$$

For actual materials and conditions these equations have been derived and justified by many authors [8-15]. All the equations for $\lambda(P, \lambda_0, \lambda_i)$ express the relation between the coefficient of thermal conductivity and one parameter of the substance, the porosity. The inclusion of a second parameter, for example, the grain dimension d , even for the simple case of a continuous medium with spherical impurities, leads to complex equations of the form

$$\lambda = f(P, \lambda_0, \lambda_i, d).$$

It can be imagined how complex is the calculation of other factors affecting the thermal conductivity such as the temperature and moisture content of the system. But calculations which would take into account the effect of all the factors on conductive heat conduction are as yet impossible to carry out. Evidently, it is a reasonable approach to analyze the actual conditions of each problem and establish the principle influences, neglecting second-order effects. Even more complicated are calculations to estimate the coefficients of convective and radiative heat conduction λ_C and λ_R .

The coefficients λ_T and λ_C are functions of a whole series of factors, primarily of the temperature. The form of these functions depends on the temperature gradient in the material, its rate of change, and the form of the structure of the particles and it may be explicit only in the simplest cases. In a specimen of small volume and in a specimen which is open at both ends, in a one-dimensional heat flux and at small values of the temperature gradient, $\partial t/\partial x$, λ_C is small. An increase in $\partial t/\partial x$ leads to an increase in λ_C ; the more rapidly the gradient increases, the stronger is the increase in λ_C , most probably according to an

exponential law. In a vessel open at one or both ends, the pattern is more complex, and λ_c cannot be represented as an exponential function of $\partial t/\partial x$.

An important result in practice, following from experiments with air cavities, is the possibility of indicating limiting temperature differences in the layer Δt and a layer thickness h , above which thermal convection begins [16-17]. It follows from these experiments that in thick layers convection begins even at small temperature differences $\Delta t > 0.3^\circ$, in layers of thickness down to 10 mm convection begins at $\Delta t \approx 5^\circ$, while in layers of thickness less than 5 mm convection is in general absent up to temperatures approaching 100° . It is interesting to note that a marked increase in the equivalent thermal conductivity occurs even in thick layers only for temperature differences less than 15° . Further increase in Δt at the edges of the layer very weakly effects the increase in the thermal conductivity. In the case of thin layers of the order of 1 mm and less, no increase in the coefficient of thermal conductivity is observed during the whole investigation in the temperature interval $\Delta t = 0-100^\circ$. Approximate preliminary calculations make it possible to estimate the proportion of heat transfer by natural convection in the total heat transport. At 40°C the percentage of the total heat transfer coming from convection is 0.13; 0.28; 1.6; and 5.5 respectively for materials composed of particles with dimensions 0.1; 0.2; 1.0; and 3.0 mm. Thus, we can establish approximately a) the linear behavior of this component of heat transfer as the dimension of the particles changes and b) the virtual absence of convection for fine-grained systems (maximum 5%). At moderate temperatures, as already explained, the definitive factor is the temperature difference. The effect of the absolute value of the temperature appears in the coefficient of radiative heat conduction λ_r . In addition, λ_r is also affected by the dimension of the pores, the degree of blackness of the radiating walls of the pores. There are a number of papers in which a theoretical approach is given to the establishment of the radiative heat conduction. Unfortunately, the great majority of the results obtained in these papers are of only a very preliminary nature in view of the arbitrariness in the initial assumptions. The latter, as a rule, amount to the possibility of replacing the actual radiating walls of the pores in the dispersive material by infinite parallel flat plates or to the assumption of an idealized structure with correctly arranged spherical grains, the thermal conductivity of which is assumed to be infinitely large by comparison with the thermal conductivity of the intermediate medium. All the known equations have to be improved since they take no account of the angular distribution of the radiation (the cosine law) and also they do not take into account sufficiently accurately the degree of blackness of the pore surfaces. These improvements, which we have undertaken, lead to the following result:

$$\lambda_r = 2e^2\sigma T^3h.$$

Rough preliminary calculations make it possible to establish the percentage of the radiation component in the total heat transfer in a dispersive material as a function of the temperature and the dimension of the particles. It follows from these calculations that the effect of the temperature on the thermal radiation appears only in materials with particles of dimension greater than 1 mm. For very large particles, of dimension 6 mm, the fraction of heat transfer by radiation does not exceed 7 per cent of the total heat transfer. The radiation has little effect on heat transfer only at low temperatures.

The rate of increase of λ_r becomes extremely intense as we move towards particles of large grain size.

The approximate nature of similar calculations is expressed in the neglect of the temperature gradient inside the grain and in other inadmissible assumptions. The fundamental defect, however, in all the possible calculations is the artificial distinction between heat transfer by conduction and by radiation. It remains to consider the effect of the last factor – moisture transport – on heat transport. The problem of estimating the degree of change in the coefficient of heat conduction of a disperse medium due to the effect of vapor transport can be discussed on the basis of papers [18-19] comparing the equation of heat conduction without a heat source and the equation complicated by the presence of a term taking into account heat transfer by vapor, $L\partial M/\partial x$, where L is the latent heat of vaporization, M is the vapor flux,

$$M = 23 \cdot 10^{-4} \frac{\varphi P \Pi L}{\mathcal{P} T^{0.7}} \cdot \frac{\partial T}{\partial x}, \text{ g/cm}^2 \cdot \text{sec}$$

Thus,

$$\lambda_m = 23 \cdot 10^{-4} \frac{\varphi P \Pi L}{\mathcal{P} T^{0.7}},$$

where φ is a coefficient defining the structure of the material, P is its porosity, Π is the water vapor pressure at temperature T , $\Pi = \mathcal{P}T_1 + \mathcal{P}T_2/2$ is the arithmetic mean of the values of the vapor pressure at the water surface at temperatures T_1 and T_2 , $\partial T/\partial x$ is the temperature drop at the surfaces of a plane-parallel layer across which there is a one-dimensional vapor flux through the temperature drop.

Analysis of the solution leads to the conclusion that while the temperature of the medium is less than 50°C , the proportion of the factor determining the heat transport by the vapor is 10% of the total effect of vapor transport. Thus, to determine the temperature field in disperse systems, the unusual, complex equation of heat conduction can be used only at high temperatures and temperature differences, when there is considerable overhumidification, and when the fine-grained nature of the material is clearly expressed.

Indeed, in solving many actual problems to determine the temperature fields in dispersive media by the method described above, we have in fact to measure $\lambda_i(x, \tau)$ and $c_i(x, \tau)$ in direct experiments using instruments. For this purpose we have developed a large number of probes for measuring the field profiles of the thermophysical characteristics [20]. In addition, such properties of the soil as the moisture content w , the density ρ , the mechanical composition of the soil d (d is the grain dimension), experience initial changes with temperature through the depth of the layer. None of these variables, w , d , ρ , occur in the initial differential equations for effective heat conduction. Hence, in addition to direct depth and temporal probing to obtain the behavior of $\lambda_i(x, \tau)$ and $c_i(x, \tau)$ in field conditions, we have to carry out an immense amount of laboratory work on the materials under investigation, taken from various soil depths; for this we have studied simultaneously relations of two types

$$\lambda_i(w, \rho, d) \text{ and } c_i(w, \rho, d),$$

$$w = f(x, \tau), \quad \rho = f''(x, \tau), \quad d = f'''(x, \tau).$$

Only after this do we obtain the equations $\lambda_i = \varphi_1(x, \tau)$; $c_i = \varphi_2(x, \tau)$ required for substitution in the initial relations (1)-(4).

This has been done for not less than 20 soil types. Finally, substituting the field or laboratory profile and temporal relations for the thermophysical characteristics, expressed in analytic form in Eq. (1), and solving the equation for various boundary conditions, we find it possible to implement the method of the field of thermal characteristics on a solid base and thus reflect the actual pattern of the thermal state of soils or any other dispersive object.

Particularly in this direction there have arisen a number of solutions which can be used in practice in agricultural physics, soil science, physical hydrology, etc. These problems include heat improvement measures; intrasoil condensation; conditions for the safe freezing of soils; the use of furrow ridge planting of plants, the effect of soil compacting or cultivation; calculation of the heat supply of soil; engineering calculations on the protection of soils against excess evaporation, etc.

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